SyDe312 (Winter 2005): Unit 5 - Solutions

Chapter 8 Ordinary Differential Equations

Problem 8.2 - 1

a)

h	x	Error	Rel. Err.	Ratio
0.2	3.0	-2.33E-2	-1.86E-2	
	6.0	-9.70E-3	-6.90E-3	
	9.0	-5.34E-3	-3.65E-3	
0.1	3.0	-1.16E-2	-9.26E-3	2.01
	6.0	-4.86E-3	-3.46E-3	2.00
	9.0	-2.68E-3	-1.83E-3	1.99
0.05	3.0	-5.77E-3	-4.62E-3	2.01
	6.0	-2.44E-3	-1.73E-3	1.99
	9.0	-1.34E-3	-9.18E-5	2.00

b)

h	x	Error	Rel. Err.	Ratio
0.2	3.0	1.1E-3	3.7E-3	
	6.0	1.7E-3	1.0E-2	
	9.0	9.4E-4	8.6E-3	
0.1	3.0	2.3E-4	7.7E-4	4.83
	6.0	8.1E-4	5.0E-3	2.04
	9.0	4.7E-4	4.3E-3	2.00
0.05	3.0	3.2E-5	1.1E-4	7.30
	6.0	4.0E-4	2.5E-3	2.03
	9.0	2.4E-4	2.1E-3	2.00

d)

h	x	Error	Rel. Err.	Ratio
0.2	3.0	2.6E-2	7.8E-2	
	5.0	1.3E-2	6.6E-2	
	7.0	8.1E-3	5.7E-2	
	9.0	5.5E-3	5.0E-2	
0.1	3.0	1.3E-2	3.8E-2	2.00
	5.0	6.5E-3	3.3E-2	2.00
	7.0	4.0E-3	2.8E-2	2.02
	9.0	2.7E-3	2.5E-2	2.03
0.05	3.0	6.2E-3	1.9E-2	2.09
	5.0	3.2E-3	1.6E-2	2.03
	7.0	2.0E-3	1.4E-2	2.00
	9.0	1.4E-3	1.2E-2	1.92

g)

h	x	Error	Rel. Err.	Ratio
0.2	3.0	-3.8E-3	1.2E-1	
	6.0	-3.6E-4	8.1E-2	
	9.0	-8.0E-5	5.9E-2	
0.1	3.0	-2.0E-3	6.1E-2	1.90
	6.0	-1.9E-4	4.2E-2	1.89
	9.0	-4.1E-5	3.0E-2	1.95
0.05	3.0	-1.0E-3	3.1E-2	2.00
	6.0	-9.6E-5	2.6E-2	1.98
	9.0	-2.1E-5	1.5E-2	1.95

Problem 8.5 - 10

b)

h	x	Error	Ratio	$y_h(x)$	$\tilde{y_h}(x) - y_h(x)$
0.2	3	3.36E-6			
	6	1.33E-7			
	9	1.71E-8			
0.1	3	1.76E-7	19.1		
	6	6.09E-9	21.8		
	9	6.44E-10	26.6		
0.05	3	1.00E-8	17.6	0.299999990	1.10E-8
	6	3.23E-10	18.9	0.162162162	3.85E-10
	9	2.90E-11	22.2	0.109756098	4.07E-11

d)

h	x	Error	Ratio	$y_h(x)$	$\tilde{y_h}(x) - y_h(x)$
0.2	3	-2.17E-6			
	6	-5.62E-7			
	9	-2.51E-7			
0.1	3	-1.46E-7	14.9		
	6	-3.77E-8	14.9		
	9	-1.68E-8	14.9		
0.05	3	-9.25E-9	15.8	0.333333343	-9.10E-9
	6	-2.39E-9	15.8	0.166666669	-2.35E-9
	9	-1.07E-9	15.7	0.111111112	-2.05E-9

Problem 8.5 - 11

To change the error tolerances, we need to change the options in MATLAB using the command line as follows:

```
>>options=odeset('RelTol',1e-5,'AbsTol',1e-5)
```

Then we use the command line as below to obtain the solution:

```
>>[x,y]=ode45('odefun',xspan,y0,options)
```

with `'odefun'` the name of the procedure computing the differential equation function. Also the number of function evaluations of the derivative to reach $x =$ final endpoint can be obtained using the commands:

```
>>options=odeset('RelTol',1e-5,'AbsTol',1e-5,'stats','on')
>>[x,y]=ode45('odefun',[x0,xFinal],y0,options)
```

b)

x	$y_h(x)$	$Y(x) - y_h(x)$	Rel. Err.
2.0	0.400000536	-0.54E-6	-1.34E-6
4.0	0.235295971	-1.85E-6	-7.88E-6
6.0	0.162162407	-0.25E-6	-1.51E-6
8.0	0.123077743	-0.82E-6	-6.67E-6
10.0	0.099010469	-0.57E-6	-5.73E-6

In this problem, the routine solves at 77 points to obtain the numerical solution with the required accuracy. The number of function evaluations to reach $x = 10$ is 121.

d)

x	$y_h(x)$	$Y(x) - y_h(x)$	Rel. Err.
2.0	0.500000726	-0.73E-6	-0.15E-5
4.0	0.250001853	-1.85E-6	-0.74E-5
6.0	0.166668676	-2.01E-6	-1.21E-5
8.0	0.125001309	-1.31E-6	-1.05E-5
10.0	0.100000848	-0.85E-6	-0.85E-5

On the interval $[1, 10]$ the routine solves at 61 points to obtain the numerical solution with the required accuracy. The number of function evalutions to reach $x = 10$ is 91.

Problem 8.5 - 12

This is done similarly to problem 11 except that we use the following function:

```
>>[x,y]=ode23('odefun',xspan,y0,options)
```

b)

x	$y_h(x)$	$Y(x) - y_h(x)$	Rel. Err.
2.0	0.4000031077	-3.11E-6	-7.77E-6
4.0	0.2352404690	5.36E-4	2.28E-3
6.0	0.1621364278	2.57E-4	1.59E-3
8.0	0.1230574339	1.95E-4	1.58E-3
10.0	0.0989945417	1.54E-4	1.55E-3

The number of function evaluations to reach $x = 10$ is 175.

d)

x	$y_h(x)$	$Y(x) - y_h(x)$	Rel. Err.
2.0	0.4999952592	4.74E-6	9.48E-5
4.0	0.2499941025	5.90E-5	2.36E-4
6.0	0.1666604459	6.22E-5	3.73E-4
8.0	0.1249938940	6.11E-5	4.88E-4
10.0	0.0999942391	5.76E-5	5.76E-4

The number of function evalutions to reach $x = 10$ is 151.